Photon splitting in a strongly magnetized plasma

M. V. Chistyakov, D. A. Rumyantsev

Division of Theoretical Physics,
Yaroslavl State (P.G. Demidov) University,
Sovietskaya 14, 150000 Yaroslavl, Russian Federation
E-mail: mch@uniyar.ac.ru, rda@uniyar.ac.ru

Abstract

The process of photon splitting $\gamma \to \gamma\gamma$ in a strongly magnetized medium of arbitrary temperature and chemical potential is considered. The partial amplitudes and probabilities of the photon splitting are calculated with take into account of the photon dispersion in a strong magnetic field and neutral electron-positron plasma.

1 Introduction

The process of photon splitting into two photons is a prominent example of the external active medium influence on the reactions with the elementary particle. Though it is forbidden in vacuum by charge conjugation symmetry of QED known as Furry's theorem it is allowed in the presence of an external magnetic field and/or plasma. It is remarkable, that such exotic process could play an important role in astrophysical phenomenona. It is particularly supposed, that this process could explain a peculiarity of the spectrum of the some radio pulsars [1]. There is one more interesting application of the process under consideration concerning the radio quiescence of SGRs and

AXPs, the objects associated with magnetars. Because photon splitting has no threshold, high-energy photons propagating at very small angles to the magnetic field in neutron star magnetospheres may split before reaching the threshold of pair production. Therefore, this process could suppress the production of electron-positron pairs required for a detectable radio emission [2]. It is assumed for the magnetars to have very strong magnetic fields which could exceed the critical value essentially $B_e = m^2/e \simeq 4.41 \times 10^{13}$ Gs [1] and it could reach the values up to $10^{14} - 10^{16}$ Gs [3]. Additionally, the analysis of the radiation spectrum of the some of these objects shows, that the relatively hot and dense electron-positron plasma in their environment could be presented [3].

Theoretical study of the photon splitting in magnetized plasma has a rather long history. The influence of magnetized plasma on the photon dispersion properties and its kinematics was first considered by Adler [4]. Further, the probability of the photon splitting in magnetized plasma was obtained in [5]. The modification of the photon splitting amplitude in a weakly magnetized plasma with the arbitrary temperature was considered in [6, 7].

In present paper the process of photon splitting $\gamma \to \gamma\gamma$ is investigated in the case of a strongly magnetized plasma, when the magnetic field strength B is the maximal physical parameter, namely $\sqrt{eB} \gg T$, μ, ω. Here T is the plasma temperature, μ is the chemical potential, ω is the initial photon energy.

2 Calculation of the amplitude

In this section we calculate the amplitude of process $\gamma \to \gamma\gamma$ in a strongly magnetized medium. This amplitude can be presented as the sum of the two terms

$$\mathcal{M} \;=\; \mathcal{M}_B + \mathcal{M}_{pl}, \tag{1}$$

where $\mathcal{M}_B$ – is the amplitude of process $\gamma \to \gamma\gamma$, corresponding to the contribution of magnetic field ($\mu = T = 0$). The second term in (1) corresponds to the photon coherent scattering from the real electrons and positrons of the

[1]We use natural units in which $c = \hbar = k = 1$, m is the electron mass, $e > 0$ is the elementary charge.

medium without change of their states ("forward" scattering) and with two photons emission. These contributions are depicted by Feynman diagrams in the Figure 1.

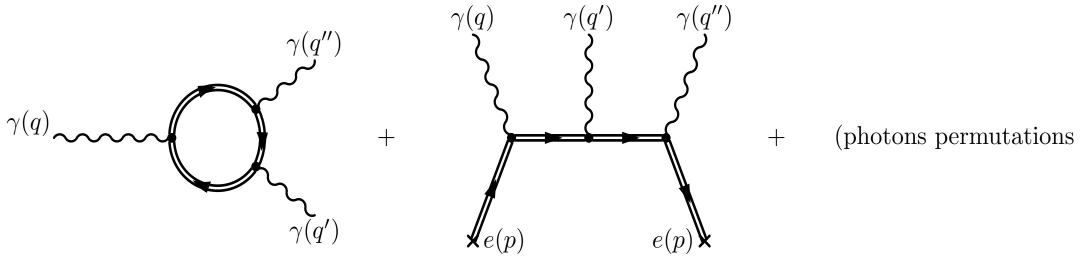

Figure 1: The Feynman diagrams for the photon splitting process in magnetized plasma. The cross at the end of the electron line symbolizes that the particle belongs to the medium.

The amplitude, corresponding to the diagrams on Figure 1 can be presented in the following form

$$\mathcal{M} = \varepsilon_\mu(q)\varepsilon_\nu(q'')\varepsilon_\rho(q')\,(\Pi^{(0)}_{\mu\nu\rho} + \Pi^{(1)}_{\mu\nu\rho}), \tag{2}$$

where

$$\begin{aligned}\Pi^{(0)}_{\mu\nu\rho} &= eB\,\frac{e^3}{4\pi^2}\frac{(\tilde\varphi q)_\mu(\tilde\varphi q'')_\nu(\tilde\varphi q')_\rho}{(q'\tilde\varphi q'')}\Big[\mathcal{J}^{(-)}_\perp(q_\parallel, q'_\parallel)\; - \\ &- \;\mathcal{J}^{(-)}_\perp(-q'_\parallel, -q_\parallel) - \mathcal{J}^{(-)}_\perp(-q''_\parallel, q'_\parallel)\; - \;(q' \leftrightarrow q'')\Big],\end{aligned} \tag{3}$$

$$\begin{aligned}\Pi^{(1)}_{\mu\nu\rho} &= -\frac{ie^3}{2\pi^2}\{(q'\varphi q'')\,[\pi_{\mu\nu\rho} + \upsilon_{\mu\nu\rho}] + (q'\mathcal{G}(q''))_\nu\,\varphi_{\rho\mu} + \\ &+ \;\frac{1}{2}((q''-q')\mathcal{G}(q))_\mu\,\varphi_{\nu\rho} + (q''\mathcal{G}(q'))_\rho\,\varphi_{\nu\mu} - \\ &- \;\mathcal{G}_{\nu\rho}(q'')\,(q'\varphi)_\mu + \mathcal{G}_{\mu\nu}(q'')\,(q\varphi)_\rho + \mathcal{G}_{\mu\rho}(q')\,(q\varphi)_\nu - \\ &- \;\mathcal{G}_{\nu\rho}(q')\,(q''\varphi)_\mu - \mathcal{G}_{\mu\nu}(q)\,(q''\varphi)_\rho - \mathcal{G}_{\mu\rho}(q)\,(q'\varphi)_\nu\; - \\ &- \;\frac{i(\tilde\varphi q)_\mu(\tilde\varphi q'')_\nu(\tilde\varphi q')_\rho}{4(q'\tilde\varphi q'')}[(q'_\perp)^2 + (q''_\perp)^2 + (q'q'')_\perp]\times \\ &\times \;[\mathcal{J}^{(-)}_\perp(q_\parallel, q'_\parallel) - \mathcal{J}^{(-)}_\perp(-q'_\parallel, -q_\parallel) - \\ &\qquad\qquad -\mathcal{J}^{(-)}_\perp(-q''_\parallel, q'_\parallel)\; - \;(q' \leftrightarrow q'')]\}.\end{aligned} \tag{4}$$

Here

$$\mathcal{G}_{\mu\nu}(q) = \left(\tilde{\Lambda}_{\mu\nu} - \frac{q_{\parallel\mu}\, q_{\parallel\nu}}{q_{\parallel}^2}\right)\,\left[H\!\left(\frac{4m^2}{q_{\parallel}^2}\right) + \mathcal{J}^{(+)}(q)\right],$$

$$\mathcal{J}^{(\pm)}(q_{\parallel}) = 2q_{\parallel}^2 m^2 \int \frac{dp_z}{E_p}\, \frac{f_-(E_p) \pm f_+(E_p)}{(q_{\parallel}^2)^2 - 4(pq)_{\parallel}^2}, \quad E_p = \sqrt{p_z^2 + m^2}, \tag{5}$$

$$\mathcal{J}_{\perp}^{(\pm)}(q_{\parallel}, q'_{\parallel}) = 2m^2 \int \frac{dp_z}{E_p}\, \frac{f_-(E_p) \pm f_+(E_p)}{[(q_{\parallel})^2 + 2(pq)_{\parallel}][(q'_{\parallel})^2 + 2(pq')_{\parallel}]}, \tag{6}$$

$f_{\pm}(E_p) = (\exp{(E_p \pm \mu)/T} + 1)^{-1}$ is the electrons (positrons) distribution functions.

$$H(z) \;=\; \frac{z}{\sqrt{z-1}}\arctan\frac{1}{\sqrt{z-1}} - 1,\; z \geq 1. \tag{7}$$

The four-vectors with indices $\perp$ and $\parallel$ belong to the Euclidean $\{1,\, 2\}$-subspase and the Minkowski $\{0,\, 3\}$-subspase correspondingly in the frame were the magnetic field is directed along third axis; $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ and $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\mu\nu}\varphi_{\mu\nu}$ are the dimensionless field tensor and dual field tensor correspondingly. The tensors $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta}$, $\widetilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta}$, with equation $\widetilde{\Lambda}_{\alpha\beta} - \Lambda_{\alpha\beta} = g_{\alpha\beta} = diag(1,-1,-1,-1)$ are introduced.

The expression for $\pi_{\mu\nu\rho}$ can be presented in the following form

$$\begin{aligned}\pi_{\mu\nu\rho} =\; & \frac{1}{q_{\parallel}^2 q_{\parallel}'^2 q_{\parallel}''^2}\Big[(q'\tilde{\varphi}q'')\big\{(\tilde{\varphi}q)_{\mu}(\tilde{\varphi}q'')_{\nu}(\tilde{\varphi}q')_{\rho}\pi_{\perp} + (\tilde{\varphi}q)_{\mu}(\widetilde{\Lambda}q'')_{\nu}(\widetilde{\Lambda}q')_{\rho}H \\ & -(\widetilde{\Lambda}q)_{\mu}(\tilde{\varphi}q'')_{\nu}(\widetilde{\Lambda}q')_{\rho}H'' - (\widetilde{\Lambda}q)_{\mu}(\widetilde{\Lambda}q'')_{\nu}(\tilde{\varphi}q')_{\rho}H'\big\} \\ & +(q'q'')_{\parallel}(\widetilde{\Lambda}q)_{\mu}(\tilde{\varphi}q'')_{\nu}(\tilde{\varphi}q')_{\rho}(H''-H') \\ & +(qq'')_{\parallel}(\tilde{\varphi}q)_{\mu}(\tilde{\varphi}q'')_{\nu}(\widetilde{\Lambda}q')_{\rho}(H-H'') \\ & +(qq')_{\parallel}(\tilde{\varphi}q)_{\mu}(\widetilde{\Lambda}q'')_{\nu}(\tilde{\varphi}q')_{\rho}(H'-H)\Big],\end{aligned} \tag{8}$$

$$\begin{aligned}\pi_{\perp} &= H' + H'' + H \\ &+2\frac{q_{\parallel}^2 q_{\parallel}'^2 q_{\parallel}''^2 - 2m^2[q_{\parallel}^2(q'q'')_{\parallel}H - q_{\parallel}'^2(qq'')_{\parallel}H' - q_{\parallel}''^2(qq')_{\parallel}H'']}{q_{\parallel}^2 q_{\parallel}'^2 q_{\parallel}''^2 - 4m^2[q_{\parallel}'^2 q_{\parallel}''^2 - (q'q'')_{\parallel}^2]},\end{aligned} \tag{9}$$

here, e.g. $H' \equiv H(4m^2/q_\parallel'^2)$,

$$\upsilon_{\mu\nu\rho} = \pi_{\mu\nu\rho}\left[\pi_\perp \to \upsilon_\perp, H \to \mathcal{J}^{(+)}(q_\parallel), H' \to \right.$$
$$\left. \to \mathcal{J}^{(+)}(q'_\parallel), H'' \to \mathcal{J}^{(+)}(q''_\parallel)\right],$$

$$\upsilon_\perp = \frac{1}{(q'\tilde{\varphi}q'')^2}\Big\{(qq')_\parallel (qq'')_\parallel \mathcal{J}^{(+)}(q_\parallel) - (qq')_\parallel (q'q'')_\parallel \mathcal{J}^{(+)}(q'_\parallel) -$$

$$- \quad (qq'')_\parallel (q'q'nonumber \tag{10}$$

1'$_\parallel$) + $\frac{q_\parallel^2 q_\parallel'^2 q_\parallel''^2}{4}\Big[\mathcal{J}_\perp^{(+)}(q_\parallel, q'_\parallel)$ +

$+\mathcal{J}_\perp^{(+)}(-q'_\parallel, -q_\parallel) + \mathcal{J}_\perp^{(+)}(-q''_\parallel, q'_\parallel) + (q' \leftrightarrow q'')\Big]\Big\}$.

3 Kinematics of photon splitting

The kinematics of the process is defined by the vacuum polarization and the photon coherent scattering from the real electrons and positrons of the plasma. The eigenvalues of the photon polarization operator in the case of neutral ($\mu = 0$) electron-positron plasma could be obtained from paper [8] and they could be presented in the following form

$$\mathcal{P}^{(1)}(q) \simeq -\frac{\alpha}{3\pi}\, q_\perp^2 - q^2\,\Lambda(B), \tag{10}$$

$$\mathcal{P}^{(2)}(q) \simeq -\frac{2eB\alpha}{\pi}\left[H\left(\frac{4m^2}{q_\parallel^2}\right) + \mathcal{J}^{(+)}(q_\parallel)\right] - q^2\,\Lambda(B), \tag{11}$$

$$\mathcal{P}^{(3)}(q) \simeq -\,q^2\,\Lambda(B), \tag{12}$$

where

$$\Lambda(B) = \frac{\alpha}{3\pi}\left[1.792 - \ln(B/B_e)\right].$$

From the analysis of the dispersion equations solutions

$$q^2 - \mathcal{P}^{(\lambda)} = 0 \qquad (\lambda = 1, 2, 3), \tag{13}$$

it is followed, that only two of them (with $\lambda = 1, 2$) correspond to the real photons with the polarization vectors

$$\varepsilon_\alpha^{(1)} = \frac{(q\varphi)_\alpha}{\sqrt{q_\perp^2}}, \qquad \varepsilon_\alpha^{(2)} = \frac{(q\tilde{\varphi})_\alpha}{\sqrt{q_\parallel^2}}. \tag{14}$$

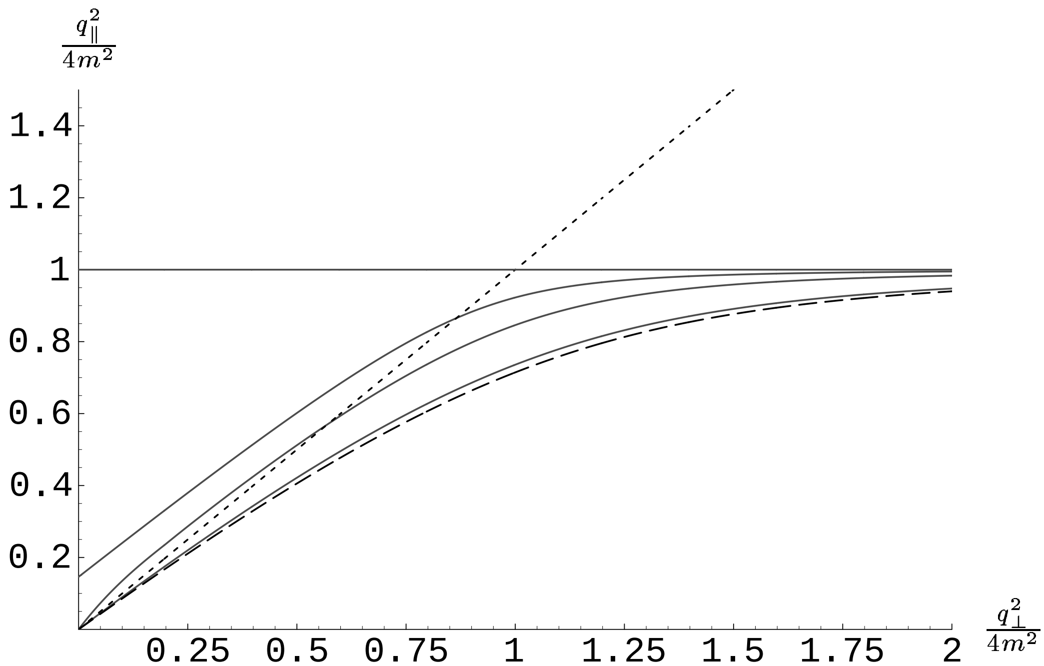

Figure 2: Photon dispersion laws in strong magnetic field $B/B_e = 200$ and neutral plasma ($T = 1\text{MeV}$, $\mu = 0$) for $q_z^2/4m^2 = 0$ (is upper curve), $q_z^2/4m^2 = 3$ (is middle curve), $q_z^2/4m^2 = 50$ (is lower curve). Photon dispersion without plasma is depicted by dashed line. Dotted line corresponds to the vacuum dispersion law, $q^2 = 0$.

Notice, that contrary to the pure magnetic field case, the mode 2 photon can have a positive value of q^2 in the kinematical region $q_\parallel^2 \le 4m^2$. Then the new channel $\gamma_2 \to \gamma_1\gamma_1$, forbidden in the magnetic field without plasma, is possible in this region. At the same time, the splitting channels $\gamma_1 \to \gamma_2\gamma_2$ and $\gamma_1 \to \gamma_1\gamma_2$ allowed in the pure magnetic field [4], are kinematically forbidden in this region.

As it follows from (11), the eigenvalue of the polarization operator $\mathcal{P}^{(2)}$ has singular behavior in the vicinity of the pair-creation threshold. It leads to the necessity of taking into account of a wave-function renormalization for the photon of mode 2

$$\varepsilon_\alpha^{(2)} \to \varepsilon_\alpha^{(2)} \sqrt{Z_2}, \quad Z_2^{-1} = 1 - \frac{\partial \mathcal{P}^{(2)}}{\partial \omega^2}. \tag{15}$$

The partial amplitudes for the channels $\gamma_1 \to \gamma_2\gamma_2$, $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_2 \to$

$\gamma_1\gamma_1$ could be obtained from (2). They could be presented in the following form

$$\mathcal{M}_{112} = i4\pi \left(\frac{\alpha}{\pi}\right)^{(3/2)} \frac{(q'\varphi q'')(q'\tilde{\varphi} q'')}{[(q')^2_\perp (q'')^2_\parallel q^2_\perp]^{(1/2)}} \left[H\left(\frac{4m^2}{(q'')^2_\parallel}\right) + \mathcal{J}^{(+)}(q''_\parallel)\right], \quad (16)$$

$$\begin{aligned}\mathcal{M}_{122} &= i4\pi \left(\frac{\alpha}{\pi}\right)^{(3/2)} \frac{(q'\tilde{\Lambda} q'')}{[(q')^2_\parallel (q'')^2_\parallel q^2_\perp]^{(1/2)}} \times \\ &\times \left\{(q\Lambda q'')\left[H\left(\frac{4m^2}{(q')^2_\parallel}\right) + \mathcal{J}^{(+)}(q'_\parallel)\right] + \right. \\ &\left. +(q\Lambda q')\left[H\left(\frac{4m^2}{(q'')^2_\parallel}\right) + \mathcal{J}^{(+)}(q''_\parallel)\right]\right\}, \quad (17) \\ \mathcal{M}_{211} &= \mathcal{M}_{112}(q \leftrightarrow q''). \quad (18)\end{aligned}$$

4 The probability of the photon splitting in a strongly magnetized plasma

The general expression for the splitting probability can be written in the form

$$\begin{aligned} W_{\lambda\to\lambda'\lambda''} &= \frac{g}{32\pi^2\omega_\lambda}\int \\ &\mid \mathcal{M}_{\lambda\to\lambda'\lambda''} \mid^2 Z_\lambda Z_{\lambda'} Z_{\lambda''} \times \times (1+f_{\omega'})(1+f_{\omega''}) \\ &\delta(\omega_\lambda(\mathbf{k}) - \omega_{\lambda'}(\mathbf{k}-\mathbf{k}'') - \omega_{\lambda''}(\mathbf{k}''))\frac{d^3k''}{\omega_{\lambda'}\omega_{\lambda''}}, \quad (19)\end{aligned}$$

where $f_\omega = (e^{\omega/T} - 1)^{-1}$ is the photons distribution function, the factor $g = 1 - (1/2)\delta_{\lambda'\lambda''}$ is inserted to account for the possible identity of the final photons.

It is clear from the formula (19) that the calculation of the splitting probability presents a rather complicated mathematical problem. The simple expressions for the probabilities of the channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ can be obtained in the asymptotic limit $m^2 \ll \omega^2 \sin^2\theta \le eB$:

$$W_{1\to 12} \simeq \frac{\alpha^3 T^2}{4\omega \sin^2\theta}\left[(1 - u)^2 F\left(\frac{\omega(1+u)}{2T}\right) + (u \to -u)\right], \quad (20)$$

where $u = \cos\theta$, θ is the angle between the initial photon momentum $\mathbf{k}$ and the magnetic field direction $\mathbf{B}$,

$$F(z) = \int\limits_0^z \frac{\mathcal{F}^2(x/2)\, dx}{[1 - \exp(-x)]\,[1 - \exp(x - \omega/T)]},$$

$$\mathcal{F}(x) = \frac{\sinh x}{\cosh x + 1}.$$

$$W_{1\to 22} \simeq \frac{\alpha^3 m^2}{4\omega} \frac{1}{1 - \exp\left[-\frac{\omega}{T}(1-u)\right]} \frac{1}{1 - \exp\left[-\frac{\omega}{T}(1+u)\right]} \times$$
$$\times \left\{\mathcal{F}^2\left[\frac{\omega}{4T}(1-u)\right] + (u \to -u)\right\}. \tag{21}$$

The probability of channel $\gamma_2 \to \gamma_1\gamma_1$ can be simplified in the case of the rare photon gas ($f_{\omega'} = f_{\omega''} = 0$):

$$W_{2\to 11} \simeq \frac{\alpha^3}{8\pi^2} Z_2 \left(H\!\left(\frac{4m^2}{q_\parallel^2}\right) + \mathcal{J}^{(+)}(q_\parallel) \right)^2 \frac{q_\perp^2}{\omega}\, f\left(\frac{q_\parallel^2}{q_\perp^2}\right) \Theta(q^2), \tag{22}$$

where $f(x) = \ln x - 1 + 1/x$ and $\Theta(x)$ is theta-function.

We also have made the numerical calculations of the photon splitting probabilities in the case of the initial photon propagation across the magnetic field direction for the channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_2 \to \gamma_1\gamma_1$. Our results are represented in Figures 3 and 4.

5 Conclusions

In conclusion, we have investigated the process of the photon splitting in the presence of strongly magnetized plasma. The partial amplitudes and polarizations selection rules were obtained. The probabilities corresponding to the allowed channel were calculated with taking into account of the photon dispersion and wave-function renormalization. The obtaining results show, that the plasma influence modifies the polarization selection rules in comparison with pure magnetic field. In particular, the new splitting channel $\gamma_2 \to \gamma_1\gamma_1$, forbidden without plasma, is allowed. On other hand, as it is follow from

Figure 3: The dependence of the probability of channel $\gamma_1 \to \gamma_1\gamma_2$ on energy in a strong magnetic field $B/B_e = 200$ and neutral ($T = 1$MeV, $\mu = 0$) plasma. The dashed line corresponds to the probability in the pure magnetic field ($T = \mu = 0$) [9]. The asymptotics (20) is depicted by dots. Here $W_0 = (\alpha/\pi)^3 m$.

the formulas (20), (21) and numerical calculations, Figure 3, the presence of plasma suppresses the probabilities of channels $\gamma_1 \to \gamma_1\gamma_2$ and $\gamma_1 \to \gamma_2\gamma_2$ in comparison with pure magnetic field. As a result, it could lead to the modification in the mechanism of the spectra formation of SGR and AXP.

Acknowledgements

We express our deep gratitude to the organizers of the Seminar "Quarks-2004" for warm hospitality.

This work supported in part by the Council on Grants by the President of Russian Federation for the Support of Young Russian Scientists and Leading Scientific Schools of Russian Federation under the Grant No. NSh-1916.2003.2, by the Russian Foundation for Basic Research under the Grant No. 04-02-16253, and by the Ministry of Education of Russian Federation under the Grant No. E02-11.0-48.

References

[1] A.C. Harding, M.G. Baring, P.L. Gonthier, Astrophys.J. **476**, 246 (1997).

[2] M.G. Baring, A.C. Harding, Astrophys. J. Lett. **507**, L55 (1998).

[3] R. C. Duncan, C. Thompson, Astrophys. J. **392**, L9 1992.

[4] S.L. Adler, Ann. Phys. (N.Y.) **67**, 599 (1971).

[5] T. Bulik, Acta Astronomica. **48**, 695 (1998).

[6] P. Elmfors, B. Skagerstam, Phys. Lett. **B427**, 197 (1998).

[7] J. M. Martinez Resco, M. A. Valle Basagoiti, Phys. Rev. **D64**, 016006 (2001).

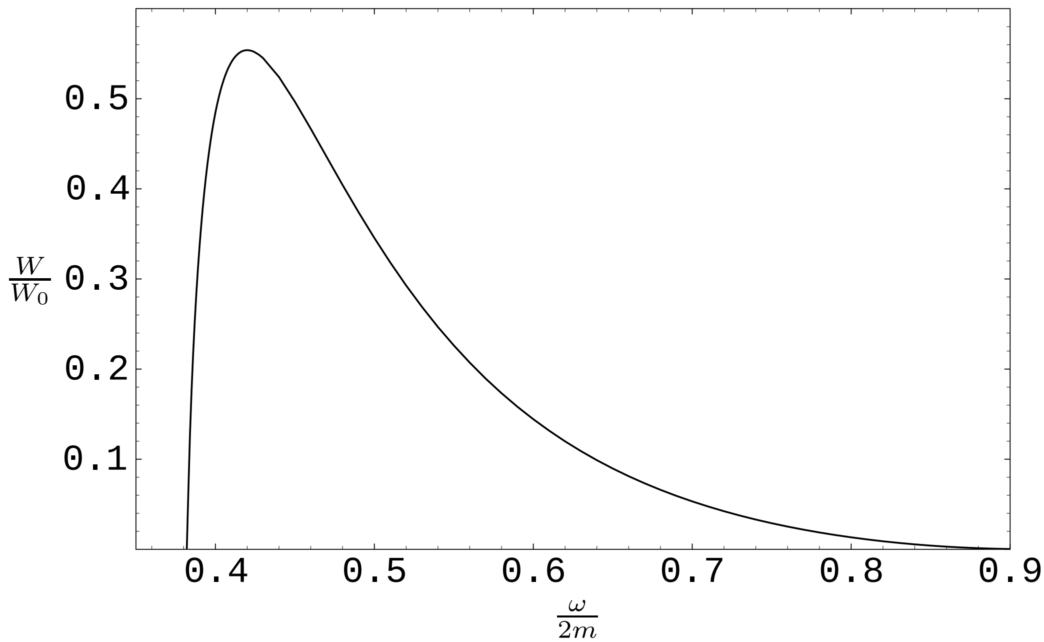

Figure 4: The dependence of the probability of channel $\gamma_2 \to \gamma_1\gamma_1$ on energy in a strong magnetic field $B/B_e = 200$ and neutral ($T = 1$MeV, $\mu = 0$) plasma.

[8] A.E. Shabad, Tr. Fiz. Inst. Akad. Nauk SSSR **192**, 5 (1988).

[9] M.V. Chistyakov, A.V. Kuznetsov, N.V. Mikheev, Phys.Lett. **B434**, 67 (1998).