Neutrino heating of a shock wave within magnetorotational model

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Abstract

Based on the magnetorotational model of a supernova explosion with core collapse, we investigate the significant processes of neutrino heating of the supernova shock. These processes should be taken into account in self-consistent modeling, since the neutrino heating mechanism is capable of increasing the explosion efficiency. We show that, even in the presence of a strong magnetic field ($B \sim 10^{15}$ G) in the shock formation region, the heating rate is determined with good accuracy by the absorption and emission of neutrinos in direct URCA processes. Moreover, the influence on them of a magnetic field is reduced to insignificant corrections.

Explosions of supernovae with core collapse are known to be generally accompanied by an intense outward ejection of part of the material. However, an efficient explosion do not occur in the framework of the currently existing models. Thus, for example, in the standard spherically symmetric supernova explosion model, the shock stops on a scale on the order of a hundred kilometers from the center of the remnant. Attempts to improve this model by applying relativistic corrections and using a self-consistent description of neutrino propagation (based on the solution of the Boltzmann equation) do not lead to a significant modification of the explosion pattern (Liebendoerfer et al. 2001). The currently existing 2D calculations including the additional shock heating through convection and interaction with the neutrino flux do not lead to a successful supernova explosion either (Buras et al. 2003). On the other hand, the currently available observational data on several supernovae suggest that their explosions are asymmetric (Wheeler et al. 2002; Wang et al. 2001); moreover, this asymmetry can be relatively large (Leonard et al. 2000). It would be natural to assume that this asymmetry is the result of the rapid rotation of the collapse remnant or the presence of a strong magnetic field. Note that, according to existing models, the generation of a magnetic field in the remnant is directly related to its rapid rotation.

At present, the best-known supernova explosion model with a self-consistent allowance for the magnetic field is the so-called magnetorotational model by Bisnovatyi-Kogan (1970). The presence of a primary magnetic field and an angular velocity gradient in this model leads to the linear growth of a secondary magnetic field with time to a certain critical value. Once the latter has been reached, an axially symmetric (relative to the equatorial plane) supernova explosion occurs. However, as recent calculations by Ardeljan et al. (2004) showed, the linear growth of the magnetic field is disrupted by the development of magnetorotational instability. The development of this instability leads to a rapid growth of magnetic field perturbations to strengths $B \sim 10^{15} - 10^{16}$ G, and to the formation of a shock.

As the magnetorotational instability develops, the kinetic energy of the rotation of the envelope with an angular velocity gradient transforms into the kinetic energy of the outward

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ejection of material through the rapidly growing magnetic field perturbations (Balbus and Hawley 1991, 1998). However, another additional energy source, shock heating by a neutrino flux, has long been known (Bethe and Wilson 1985). In the model under consideration, the neutrino heating mechanism is capable of increasing the explosion efficiency and is of particular interest.

The direct URCA processes

$$\nu_e + n \to p + e^-, \tag{1}$$
$$\tilde{\nu}_e + p \to n + e^+. \tag{2}$$

are generally believed to be the dominant neutrino shock heating reactions. Another popular neutrino-lepton process,

$$\nu_i + \tilde{\nu}_i \to e^+ + e^-, \quad (i = e, \mu, \tau), \tag{3}$$

is inefficient far from the center, because the angle between the neutrino and antineutrino momenta is small. Note that in a medium with a strong magnetic field, the production processes of an e^+e^- pair by a single neutrino,

$$\nu_i \to \nu_i + e^+ + e^-, \tag{4}$$
$$\tilde{\nu}_i \to \tilde{\nu}_i + e^+ + e^-, \tag{5}$$
$$(i = e, \mu, \tau).$$

open up kinematically and can be important. In this paper, we compare the neutrino shock heating efficiencies in the presence of a strong magnetic field in the standard direct URCA processes and reactions (4)-(5).

The neutrino heating rate per nucleon in the direct URCA processes (1)-(2) can be calculated as

$$Q_0^{\nu,\tilde{\nu}} = \frac{1}{N_N}\int \omega K_{\nu,\tilde{\nu}} f_{\nu,\tilde{\nu}}(\omega,\vec{r})\frac{d^3k}{(2\pi)^3}, \tag{6}$$

where $k_\alpha = (\omega, \vec{k})$ is 4-momentum of the (anti)neutrinos, $f_{\nu,\tilde{\nu}}(\omega,\vec{r})$ is their local distribution function, N_N is the local nucleon number density, and $K_{\nu,\tilde{\nu}}$ is the absorption coefficient defined as the rate of reactions (1)-(2) underintegrated over the neutrinos. In what follows, we use the natural system of units with $c = \hbar = k = 1$.

In the case of a moderate magnetic field where the e^+e^- plasma occupies many Landau levels ($\langle\omega^2_{\nu_e}\rangle \gtrsim 2eB$), its influence on the direct URCA processes is rather weak. For the absorption coefficient, we can in this case use its field-free expression. Assuming that the e^+e^- plasma is ultrarelativistic and that the nonrelativistic nucleons have a Boltzmann distribution, we can represent the absorption coefficient as

$$K_{\nu,\tilde{\nu}} = \frac{G^2}{\pi}(1+3g_a^2)Y_{n,p}N_N\,\frac{\omega^2}{1+\exp[(-\omega \pm \mu_e)/T]}. \tag{7}$$

Here, $G = G_F\cos\theta_c$, where G_F is the Fermi constant, θ_c is the Cabibbo angle, $g_a \simeq 1.26$ is the axial constant of the charged nucleon current, $Y_n = N_n/N_N$, $Y_p = 1 - Y_n$, $N_N = N_n + N_p$, $\Delta = m_n - m_p$, where N_n, N_p, m_n, m_p are the local neutron and proton number densities and masses, respectively, μ_e is the chemical potential of the electrons, and T is the local temperature.

It is convenient to represent formula (6) for the heating rate of the medium in the direct URCA processes in terms of the mean quantities of neutrino radiation:

$$\langle\omega^n_{\nu_e}\rangle = \left(\int \omega^{n+1} f_{\nu_e}(\omega,\vec{r})\frac{d^3k}{(2\pi)^3}\right)\left(\int \omega f_{\nu_e}(\omega,\vec{r})\frac{d^3k}{(2\pi)^3}\right)^{-1}, \tag{8}$$

$$\langle\chi_{\nu_e}\rangle = \left(\int \chi\omega f_{\nu_e}(\omega,\vec{r})\frac{d^3k}{(2\pi)^3}\right)\left(\int \omega f_{\nu_e}(\omega,\vec{r})\frac{d^3k}{(2\pi)^3}\right)^{-1}, \tag{9}$$

(where χ is the cosine of the angle between the neutrino momentum and the radial direction) and the total neutrino luminosity

$$L_{\nu_e} = 4\pi r^2 \int \chi \omega f_{\nu_e}(\omega, \vec{r}) \frac{d^3 k}{(2\pi)^3}. \tag{10}$$

Here r is the distance from the center of the remnant to a given point.

Under the additional simplifying assumption

$$\langle \omega^2_{\nu_e} \rangle L_{\nu_e} / \langle \chi_{\nu_e} \rangle = \langle \omega^2_{\tilde{\nu}_e} \rangle L_{\tilde{\nu}_e} / \langle \chi_{\tilde{\nu}_e} \rangle ,$$

which holds for various supernova explosion models, we obtain a well-known (see, e.g., Janka 2001) expression for the shock heating rate in the direct URCA processes:

$$\begin{aligned} Q_0 &= Q_0^{\nu} + Q_0^{\tilde{\nu}} = \frac{G^2}{\pi}(1 + 3g_a^2) \frac{L_{\nu_e} \langle \omega^2_{\nu_e} \rangle}{4\pi r^2 \langle \chi_{\nu_e} \rangle} \simeq \\ &\simeq 55 \left(\frac{MeV}{s \cdot nucleon} \right) \left(\frac{L_{\nu_e}}{10^{52} erg/s} \right) \left(\frac{\langle \omega^2_{\nu_e} \rangle}{225 MeV^2} \right) \left(\frac{10^7 cm}{r} \right)^2 , \end{aligned} \tag{11}$$

where r is the characteristic distance to the shock.

The heating rate of the medium per nucleon in the additional processes (4)-(5) can be calculated as

$$Q_B^{\nu_i, \tilde{\nu}_i} = \frac{1}{N_N} \int E_{\nu_i, \tilde{\nu}_i} f_{\nu_i, \tilde{\nu}_i}(\omega, \vec{r}) \frac{d^3 k}{(2\pi)^3}, \tag{12}$$

where $E_{\nu_i, \tilde{\nu}_i}$ is the heating rate of the medium per (anti)neutrino of type i. In the case of a moderate magnetic field where $\langle \omega^2_{\nu_i} \rangle \gg m_e^2 \gtrsim eB$, it can be represented in a logarithmic approximation (Kuznetsov and Mikheev 1997) as

$$E_{\nu_i, \tilde{\nu}_i} \simeq \frac{7 G_F^2 (c_{v_i}^2 + c_{a_i}^2)}{432 \pi^3} (eB \omega_{\nu_i} \sin\varphi)^2 \ln \left(\frac{eB \omega_{\nu_i} \sin\varphi}{m_e^3} \right). \tag{13}$$

Here, φ is the angle between the momentum of the initial neutrino and the magnetic field, c_{v_i} c_{a_i} ($c_{v_e} \simeq 0.96$, $c_{a_e} = 1/2$ for the electron neutrino; $c_{v_i} \simeq -0.04$, $c_{a_i} = -1/2$ for the μ- and τ-neutrino).

However, the magnetic field $B \gg B_0 = m_e^2/e$ can be generated in a supernova shock wave. In the case of relatively strong magnetic field where $\langle \omega^2_{\nu_i} \rangle \gg 2eB \gg m_e^2$, the heating rate was calculated as

$$E_{\nu_i, \tilde{\nu}_i} \simeq \frac{7 G_F^2 (c_{v_i}^2 + c_{a_i}^2)}{216 \pi^3} (eB \omega_{\nu_i} \sin\varphi)^2 \ln \left(\frac{\omega^2_{\nu_i} \sin^2\varphi}{eB} \right). \tag{14}$$

This formula was obtained in a logarithmic approximation also. Using this expression, formula (12) for the neutrino heating rate of the medium in processes (4)-(5) can also be expressed in terms of the mean quantities of neutrino radiation and its total luminosity:

$$Q_B^i = Q_B^{\nu_i} + Q_B^{\tilde{\nu}_i} = \frac{7 G_F^2 (c_{v_i}^2 + c_{a_i}^2)}{108 \pi^3} \frac{(eB)^2 \langle \omega_{\nu_i} \rangle}{N_N} \frac{L_{\nu_i}}{4\pi r^2 \langle \chi_{\nu_i} \rangle} \ln \left(\frac{\langle \omega^2_{\nu_i} \rangle}{eB} \right). \tag{15}$$

When deriving this formula, we assumed that $\sin\varphi \sim 1$, which is right approximately for the region where the neutrinos propagate almost freely. Under the additional simplifying assumption

$$\langle \omega_{\nu_i} \rangle L_{\nu_i} / \langle \chi_{\nu_i} \rangle = \langle \omega_{\tilde{\nu}_i} \rangle L_{\tilde{\nu}_i} / \langle \chi_{\tilde{\nu}_i} \rangle = \langle \omega_{\nu_e} \rangle L_{\nu_e} / \langle \chi_{\nu_e} \rangle ,$$

which holds good for various supernova explosion models, we obtained the following expression for the ratio of the total neutrino heating rates in processes (4)-(5) and (1)-(2):

$$\frac{Q_B}{Q_0} \simeq 1.0 \times 10^{-2} \, \frac{(eB)^2 \, \langle \omega_{\nu_e} \rangle}{N_N \, \langle \omega^2_{\nu_e} \rangle} \simeq \frac{9MeV}{\langle \omega_{\nu_e} \rangle} \, \frac{(eB)^2}{\rho}, \tag{16}$$

where $Q_B = \sum_i Q^i_B$ is the total neutrino heating rate for all types of neutrinos.

In this paper, we considered the most significant neutrino shock heating processes in the magnetorotational model. For the sake of generality, we derived the well-known expression for the heating rate in the URCA processes (11). Note that even this expression contains a number of simplifying assumptions discussed in the paper. In addition, the density of the medium decreases with distance much more slowly in the magnetorotational model than in the spherically symmetric model. This implies that, even at the characteristic distances where the shock is formed ($r \sim 100$ km), we must also take into account the neutrino radiation processes, which can significantly reduce the total rate of neutrino shock heating. Thus, formula (11) should be treated with caution, particularly in the magnetorotational model.

On the other hand, in the magnetorotational model the strong magnetic field ($B \sim 10^{15}$ G) can be generated at large distances ($r \sim 100$ km). Therefore, we must consider the effect of such a strong magnetic field on the neutrino shock heating. In particular, under these conditions, the new neutrino heating reactions (4)-(5) can compete with the direct URCA processes (1)-(2), which are the main processes in the spherically symmetric explosion model.

Our estimate (16) shows that the new neutrino heating reactions (4)-(5) become significant when $(eB)^2 \gtrsim \rho$. However, the strength of the magnetic field produced by the medium cannot be too large. For example, in the models with sub-Keplerian rotation rates (Akiyama et al. 2003; Thompson et al. 2004), the magnetic field strength reaches saturation when the field energy density becomes comparable to the rotation energy density of the medium, $B^2_{sat} \simeq 4\pi\rho(r\Omega)^2$ (where Ω is the local angular velocity of the medium at distance r). Using this estimate, we can present the ratio of the heating rates (16) as

$$\frac{Q_B}{Q_0} \lesssim 0.1 \left(\frac{r\Omega}{c} \right)^2 \frac{9MeV}{\langle \omega_{\nu_e} \rangle} \ll 1, \tag{17}$$

where c is the speed of light in a vacuum. Thus, the new reactions that open up in a magnetic field cannot compete with the standard neutrino shock heating processes. Consequently, even in the case of a strong magnetic field, $B \sim 10^{15}$ G, the heating is almost completely determined by the absorption and emission of neutrinos in the direct URCA processes, with the influence of the magnetic field on them being reduced to insignificant corrections.

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