Plasma induced fermion spin-flip conversion $f_L \to f_R + \gamma$

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Abstract

The fermion spin-flip conversion $f_L \to f_R + \gamma$ is considered, caused by the difference of the additional energies of the electroweak origin, acquired by left- and right-handed fermions (neutrino, electron) in medium. An accurate taking account of the fermion and photon dispersion in medium is shown to be important.

1 Introduction

The most important event in neutrino physics of the last decades was the solving of the Solar neutrino problem. The Sun appeared in this case as a natural laboratory for investigations of neutrino properties. There exists a number of natural laboratories, the supernova explosions, where gigantic neutrino fluxes define in fact the process energetics. It means that microscopic neutrino characteristics, such as the neutrino magnetic moment, etc., would have a crucial impact on macroscopic properties of these astrophysical events.

One of the processes caused by the photon interaction with the neutrino magnetic moment, which could play an important role in astrophysics, is the radiative neutrino spin flip transition $\nu_L \to \nu_R \gamma$. The process can be kinematically allowed in medium due to its influence on the photon dispersion, $\omega = |\mathbf{k}|/n$ (here $n \neq 1$ is the refractive index), when the medium provides the condition $n > 1$. In this case the effective photon mass squared is negative, $m_\gamma^2 \equiv q^2 < 0$. This corresponds to the well-known effect of the neutrino Cherenkov radiation [1].

There exists also such a well-known subtle effect as the additional energy W acquired by a left-handed neutrino in plasma. This additional energy was considered in the series of papers by Studenikin et al. [2] as a new kinematical possibility to allow the radiative neutrino transition $\nu_L \to \nu_R \gamma$. The effect was called the "spin light of neutrino" ($SL\nu$), and later the similar effect "spin light of electron" (SLe) was discovered. For unknown reasons, the photon dispersion in medium providing in part the photon effective mass, was ignored in these papers. However, it is evident that a kinematical analysis based on the additional neutrino or electron energy in matter (caused by the weak forces) when the matter influence on the photon dispersion (caused by electromagnetic forces) is ignored, cannot be considered as a physical approach. Similarly, in the SLe effect the authors [2] considered the matter influence on electron by the weak forces and ignored the electromagnetic interaction, taking the unphysical case of a pure neutron medium. It should be noted that even in the conditions of a cold neutron star, the fraction of electrons and protons cannot be exacly zero, $Y_e \gtrsim 0.01$ [3]. Moreover, even if this unphysical case of a pure neutron medium is considered, one should take into account the electromagnetic interaction of electrons with the magnetic moments of neutrons, which can be much more intensive than the weak interaction effects.

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A consistent analysis of the radiative neutrino spin flip transition in medium was performed in our papers [4, 5], where the medium influence both on the photon and neutrino dispersion was taken into account. It was shown that the threshold arose in the process, caused by the photon (plasmon) effective mass. This threshold left no room for the so-called "spin light of neutrino" and "spin light of electron" in the real astrophysical situations.

In the series of papers [6] the authors declare that they have developed a powerful method of exact solutions of the modified Dirac equations including the effective matter potentials.

In this paper, we remind the basic points of our criticism upon the $SL\nu$ effect, and give some comments on the method of exact solutions [6].

2 Additional left-handed neutrino energy and effective mass

As it was already mentioned, the effect of the "spin light of neutrino" proposed in [2] was based on the additional left-handed neutrino energy W induced by the medium influence. Just this additional energy provides an effective mass squared m_L^2 to the left-handed neutrino,

$$m_L^2 = \mathcal{P}^2 = (E + W)^2 - \mathbf{p}^2 = 2\,E\,W + W^2 + m_\nu^2\,, \tag{1}$$

where $\mathcal{P}$ is the neutrino four-momentum in medium, while $(E,\, \mathbf{p})$ would form the neutrino four-momentum in vacuum, $E = \sqrt{\mathbf{p}^2 + m_\nu^2}$. It will be further seen that the neutrino vacuum mass m_ν, a great attention was paid to in the $SL\nu$ analysis [2], may safely be neglected.

Given a $\nu_L \nu_R \gamma$ interaction, caused by the neutrino magnetic moment, the left-handed neutrino effective mass m_L (1) would open a kinematical possibility for the process $\nu_L \to \nu_R + \gamma$, if the photon effective mass is less than m_L.

Basing the consideration of the "spin light of neutrino" on the additional left-handed neutrino energy, the authors [2], nevertheless, did not analyse this value in detail.

The expression for this additional energy of a left-handed neutrino with the flavor $i = e, \mu, \tau$ was obtained in the local limit of the weak interaction [7–9], see also Ref. [10], and can be presented in the following form

$$\begin{aligned} W_i = \sqrt{2}\, G_{\rm F} \Bigg[& \left(\delta_{ie} - \frac{1}{2} + 2\, \sin^2\theta_{\rm W} \right) \left(N_e - \bar{N}_e \right) + \left(\frac{1}{2} - 2\, \sin^2\theta_{\rm W} \right) \left(N_p - \bar{N}_p \right) \\ & - \frac{1}{2} \left(N_n - \bar{N}_n \right) + \sum_{\ell = e, \mu, \tau} \left(1 + \delta_{i\ell} \right) \left(N_{\nu_\ell} - \bar{N}_{\nu_\ell} \right) \Bigg]\,, \end{aligned} \tag{2}$$

where the functions $N_e, N_p, N_n, N_{\nu_\ell}$ are the number densities of background electrons, protons, neutrons, and neutrinos, and $\bar{N}_e, \bar{N}_p, \bar{N}_n, \bar{N}_{\nu_\ell}$ are the densities of the corresponding antiparticles. To find the additional energy for antineutrinos, one should change the total sign in the right-hand side of Eq. (2).

As is seen from Eq. (2), this value becomes zero in the charge-symmetric plasma. This means that the local limit of the weak interaction does not describe comprehensively the additional neutrino energy in plasma, and the non-local weak contribution must be taken into account. The analysis of this contribution was first performed for the conditions of the early Universe [7, 10].

The non-local weak contribution into the additional neutrino energy in plasma, which is identical for both neutrinos and antineutrinos, can be presented in the form

$$\Delta^{(\rm nloc)} W_i = - \frac{16\, G_{\rm F}\, E}{3\, \sqrt{2}} \left[\frac{< E_{\nu_i} >}{m_Z^2} \left(N_{\nu_i} + \bar{N}_{\nu_i} \right) + \delta_{ie}\, \frac{< E_e >}{m_W^2} \left(N_e + \bar{N}_e \right) \right], \tag{3}$$

where E is the energy of a neutrino propagating through plasma, $< E_{\nu_i} >$ and $< E_e >$ are the averaged energies of plasma neutrinos and electrons correspondingly. In a particular case of a charge symmetric hot plasma, this expressions reproduces the result of Refs. [7, 10]:

$$\Delta^{(\mathrm{nloc})} W_i = -\frac{7\sqrt{2}\,\pi^2\, G_{\mathrm{F}}\, T^4}{45} \left(\frac{1}{m_Z^2} + \frac{2\,\delta_{ie}}{m_W^2} \right) E\,. \tag{4}$$

The minus sign in (4) unambiguously shows that in the early Universe the process of the radiative spin-flip transition is forbidden both for neutrinos and antineutrinos.

The absolute value of the non-local weak contribution (3) grows with the neutrino energy. It means that this contribution can be essential at ultra-high neutrino energies.

3 Does the window for the "spin light of neutrino" exist?

To show manifestly that the case considered in the papers by Studenikin et al. [2], with taking the additional left-handed neutrino energy W in plasma and ignoring the photon dispersion, was really unphysical, let us consider the region of integration for the $\nu_L \to \nu_R$ conversion width. In Fig. 1, the photon vacuum dispersion line $q_0 = k$ is inside the allowed kinematical region (left plot), but the plasma influenced photon dispersion curve appears to be outside, if the neutrino energy is not large enough (right plot).

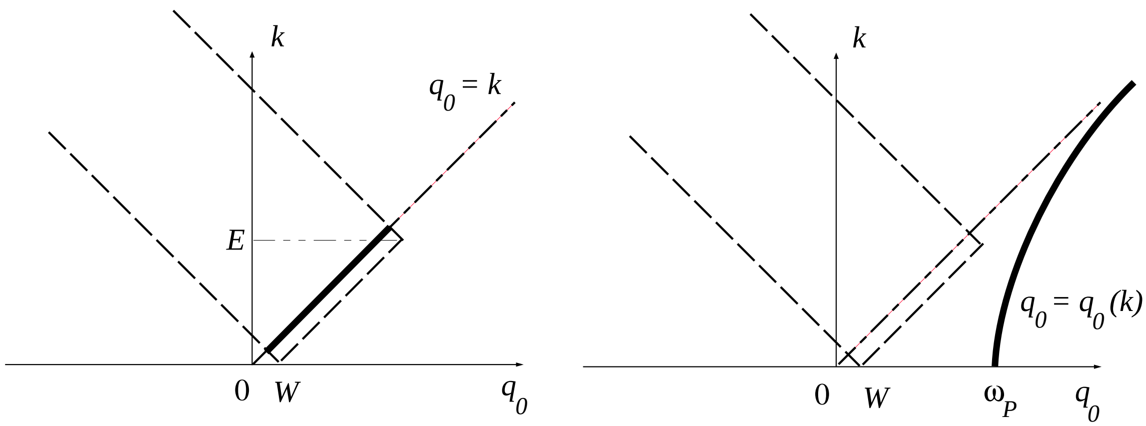

Figure 1: The region of integration for the $\nu_L \to \nu_R$ conversion width with the fixed initial neutrino energy E is inside the slanted rectangle shown by dashed line. The vacuum photon dispersion (if the medium influence is ignored) is shown by bold line in the left plot. The photon dispersion curve in plasma is shown by bold line in the right plot.

For the fixed plasma parameters, the threshold neutrino energy E_{min} exists for coming of the dispersion curve into the allowed kinematical region. Even for the interior of a neutron star this threshold neutrino energy is rather large:

$$E_{\mathrm{min}} \simeq \frac{\omega_P^2}{2\,W} \simeq 10\,\mathrm{TeV}\,, \tag{5}$$

where ω_P is the plasmon frequency.

One could hope that the "spin light of neutrino" may be possible at ultra-high neutrino energies. However, in this case the local limit of the weak interaction is incomplete, and the non-local weak contribution into additional neutrino energy W must be taken into account. This contribution has always a negative sign, and its absolute value grows with the neutrino energy. One could only hope that the window arises in the neutrino energies for the process to be kinematically opened, $E_{\mathrm{min}} < E < E_{\mathrm{max}}$. For example, in the solar interior there is no window for the process with electron neutrinos at all. A more detailed analysis of this subject was performed in our papers [4, 5].

4 "Exact solutions" of inexact equations

To construct the amplitudes of the fermion spin-flip conversion processes $f_L \to f_R + \gamma$, it is necessary to know the solutions of the Dirac equation including the matter effects. In the series of papers [6] the authors declare that they have developed a powerful method of exact solutions of the modified Dirac equations in matter.

First of all, there is some progress in the last papers [6] with respect to the $SL\nu$ effect. Namely, the five completely unphysical regions of parameters, which were discussed in details in the previous papers, are removed from the analysis, and only one case of an ultra-high neutrino energy is considered. However, an essential threshold effect is still not mentioned at all. An incorrect statement is repeated in [6] that our results [4, 5] for the case of an ultra-high neutrino energy exactly reproduce the results of the authors [2]. In fact, the width for the process $\nu_L \to \nu_R + \gamma$ obtained in our papers [4, 5] had much more general form being valid for arbitrary neutrino energies above the threshold, while the process width presented in [2] could be valid for the neutrino energies much greater than the threshold.

As in the previous papers, the non-local weak contribution into the additional neutrino energy in plasma is not taken into account in [6], while it is essential at ultra-high neutrino energies. Without this non-local weak contribution, the Dirac equation in medium for a neutrino is approximate by definition, and the term of an "exact solution" becomes dubious.

Making an attempt of constructing a new approach to the description of the neutrino and electron processes in matter, the authors [6] refer to the method of exact solutions developed for the processes in a strong external electromagnetic field. However, it is not a good justification. The strong field influence on the properties of charged particles is the essential non-perturbative effect where the analysis of the quantum processes, based just on exact solutions, is required. On the other hand, the matter influence on the neutrino and electron processes due to the weak interaction is essentially perturbative in any conceivable astrophysical conditions.

Moreover, the explicit form of the modified Dirac equation is rather simple in the low-energy approximation only, when the modification is caused by the additional left-handed neutrino energy in the form (2) calculated in the local limit of the weak interaction. As it was mentioned above, for high neutrino energies the non-local weak contribution (3) growing linearly with the neutrino energy, appears to be essential. And even this non-local term (3) is nothing but the result of the expansion of the W-boson propagator over the parameter Q^2/m_W^2. It is obvious that such an expansion has a physical sence for the neutrino energies $E \ll m_W^2/m_e \sim 10^4$ TeV. So, if one pretends to describe the additional left-handed neutrino energy in matter for the neutrino energies much higher the threshold energy (5), an exact calculation is required without expanding the W-boson propagator. In this case the modified Dirac equation should take the form of the integro-differential equation. Without such an analysis, speculations on exact solutions of the modified Dirac equation in matter have no ground.

5 Conclusion

- We have shown that an approach based on a subtle effect of the medium influence on the neutrino dispersion, when the much more significant influence of the same medium on the photon dispersion is ignored, has no physical sense.
- With the photon dispersion taken into account, the threshold neutrino energy exists for the process $\nu_L \to \nu_R + \gamma$, which is very large.
- At ultra-high neutrino energies, the non-local weak contribution into the additional neutrino energy in plasma must be taken into account. There arises the window (if exists) in the neutrino energies for the process to be kinematically opened, $E_{\min} < E < E_{\max}$.

- Without the non-local weak contribution into the additional neutrino energy in plasma, the Dirac equation in medium for a neutrino is approximate by definition, and the term of an “exact solution” becomes dubious.

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